Research Article

EDUCATIONAL POINT
An International Peer-Reviewed Journal

https://www.educationalpoint.net
ISSN: 3062-1011 (Online)

When Geometry Stays Abstract: A Qualitative Study of Pre-Tertiary TVET Instruction in Ghana

Derick Folson [1*], Francis Kwadwo Awuah [1]

[1] Kwame Nkrumah University of Science and Technology, GHANA
* Correspondence: derick.folson@knust.edu.gh

CITATION: Folson, D., & Awuah, F. K. (2026). When Geometry Stays Abstract: A Qualitative Study of Pre-Tertiary TVET Instruction in Ghana. *Educational Point*, *3*(3), e177. https://doi.org/10.71176/edup/19885

ARTICLE INFO

Received: 5 July 2026
Accepted: 18 October 2026

OPEN ACCESS

ABSTRACT

Geometry instruction in pre-tertiary Technical and Vocational Education and Training (TVET) institutions is expected to develop formal mathematical reasoning while equipping learners with transferable spatial competencies for trade practice. However, research consistently documents a persistent disconnect between abstract classroom geometry and its vocational application, particularly in Anglophone African contexts where empirical accounts of this gap remain thin. This study investigates how geometry is currently taught in Ghanaian pre-tertiary TVET institutions and the factors that facilitate or hinder abstract-practical integration. Using a qualitative, multi-method design, the study draws on fifteen structured classroom observations, and interviews with mathematics facilitators, vocational instructors, learners, and administrators across participating institutions. Findings reveal that geometry instruction remains predominantly transmission-oriented, with 80% of the fifteen observed lessons organized around lecture and direct explanation, practical activities present in only 40%, and full vocational integration observed in a single lesson (descriptive proportions of this purposive sample rather than population estimates). Integration is sustained by exceptional individual conditions, notably facilitators' prior industry experience and informal cross-disciplinary collaboration, while constrained by pervasive structural barriers, foremost among them an examination system rewarding procedural recall over applied spatial reasoning. Interpreted through Realistic Mathematics Education, Experiential Learning Theory, and Social Constructivism, these findings indicate that current instructional sequencing runs counter to principles of guided reinvention and experiential, socially mediated learning. This study provides an empirically grounded diagnostic account of the instructional and structural conditions that any future bridging framework for TVET geometry instruction in Ghana and comparable contexts must address.

Keywords: TVET mathematics, geometry instruction, abstract-practical divide, classroom observation, realistic mathematics education

Copyright © 2026 by the authors; licensee: Educational Point. This open-access article is distributed under the terms and conditions of the Creative Commons Attribution License (https://creativecommons.org/licenses/by/4.0/).

INTRODUCTION

Geometry underpins trade tasks from reading technical drawings to measuring precisely in carpentry, masonry, electrical installation, and fabrication (Acheampong et al., 2026; Battista et al., 2017), and TVET is central to national development (Davor et al., 2026). Geometry should therefore be one of the most readily contextualizable curriculum areas (Borba et al., 2017; Gbiang & Attah, 2026). Yet pre-tertiary TVET institutions across Ghana and much of Anglophone Africa continue to teach it largely as an abstract, symbol-driven subject built on definitions, theorems, and decontextualized exercises (Dalby & Noyes, 2016; Folson & Awuah, 2026; Mlambo & Sotsaka, 2025). This is linked to learner disengagement, weak transfer to technical tasks, and a view of mathematics as a generic academic requirement rather than a trade tool (Coben & Alkema, 2018). Contributing factors include curricula adapted from general education syllabi, facilitator preparation that emphasizes content mastery over contextualized pedagogy (Susilo et al., 2020), assessment that rewards procedural recall (Moore et al., 2024), and ad hoc integration that depends on individual facilitators rather than a validated framework (Mayrhofer et al., 2026; Vollet et al., 2017). This mirrors the distinction between decontextualized school mathematics and mathematics embedded in tools, materials, and tacit judgment (Dhakulkar et al., 2023; FitzSimons, 2014; Smith, 1999).

Few frameworks explicitly theorize how abstract geometric content can be systematically bridged with vocational application at the pre-tertiary level (Saripudin et al., 2023). Existing instructional models either address general pedagogy without vocational specificity or describe workshop skills without the underlying geometric reasoning (Acheampong & Awuah, 2025; Mjenda et al., 2023). Alignment studies of Ghana's TVET Core Mathematics curriculum show Coordinate Geometry and Trigonometry minimally represented in exit examinations while number and numeration dominate (Atuhurra & Kaffenberger, 2022; Folson & Awuah, 2025), but alignment alone cannot explain how geometry is actually taught and learned, or why some instructors bridge the abstract-practical divide while most do not (Folson & Awuah, 2025; Schneider et al., 2017). Existing research tends to assert this gap without documenting how it is enacted in individual lessons and stakeholder accounts (Kwateng & Leong, 2025; Vimbelo & Bayaga, 2023), and studies of inquiry-based approaches and learner perceptions (Asante-Mensa et al., 2024, 2025; Benedičič et al., 2022) leave classroom-level accounts thin. Ghana is an instructive setting because curriculum intent and assessment practice are misaligned and mathematics is taught as generic, examination-oriented content (Kwon & Lee, 2025; Zhou et al., 2022). Mathematics is embedded in measuring, cutting, spacing, and forming angles, yet the implemented curriculum rarely makes these links explicit (Borba et al., 2017; Sawyer, 2022), and prospective facilitators' geometric thinking falls short of curriculum expectations (Armah et al., 2017). This study documents lesson structure, vocational integration, content presentation, learner engagement, assessment, and resource use, and identifies the conditions that facilitate or hinder reform from the full range of stakeholders. It is interpreted through Realistic Mathematics Education (Siswantari et al., 2025; Streefland, 2012), Experiential Learning Theory, and Social Constructivism. Two research questions guide it: (1) How are geometry concepts currently taught in pre-tertiary TVET institutions in Ghana? and (2) What factors facilitate or hinder the connection between abstract geometry concepts and practical vocational applications? The findings offer a diagnostic contribution that establishes the empirical case for the framework-development work that follows.

THEORETICAL FRAMEWORK

This study is anchored in Realistic Mathematics Education (RME; Freudenthal, 1991), supplemented by Kolb's Experiential Learning Theory (Kolb, 2015) and Vygotsky's (1978) Social Constructivism. These address, respectively, how content should be structured and sequenced (Cohen et al., 2024; Tall, 2009), how learners internalize it through concrete and reflective engagement (Zhang, 2025), and the social, mediated conditions that support learning (Motseki & Joojo, 2023; Stolbova et al., 2018). Freudenthal (1991) holds that mathematics

should be taught as a human activity grounded in learners' lived reality, not as ready-made formalisms to be memorized. Learners progress through guided reinvention, moving from contextual problems through horizontal mathematization (translating a real-world problem into mathematical terms) and vertical mathematization (formalizing it within geometry's symbol system) toward general representations. A bridging framework would treat vocational tasks such as laying out a roof truss as the starting point from which concepts are reinvented, reversing the abstract-then-applied sequence.

Kolb's cycle of concrete experience, reflective observation, abstract conceptualization, and active experimentation (Kolb, 2015) supplies the lesson-level structure: learners engage in a vocational task, reflect on the spatial principles involved, formalize them, and apply them to a new problem, giving instruction a recurring structure rather than one-off contextualization (Zhang, 2025). Vygotsky's (1978) zone of proximal development holds that learners can accomplish, with a more knowledgeable other and appropriate tools, what they could not yet do alone (Motseki & Joojo, 2023; Stolbova et al., 2018). In TVET geometry, this highlights facilitator modeling, peer collaboration, and vocational tools such as set squares, levels, and technical drawings as carriers of geometric meaning (Boateng & Awuah, 2026). Together, the theories hold that abstraction should emerge from authentic vocational activity (RME), through a recurring cycle between concrete action and formal conceptualization (Kolb, 2015), socially mediated by scaffolding, peer interaction, and tool use (Vygotsky). **Table 1** operationalizes each construct as an indicator from the observation schedule or interview data and identifies the Results subsection where it is applied.

LITERATURE REVIEW

Geometry in TVET Mathematics Curricula

Geometry is foundational to technical and vocational trades, including spatial visualization, measurement, technical drawing, and material estimation (Bessot, 2015; Kyeremeh et al., 2025). Yet models of geometric thinking were developed in academic mathematics and do not address how reasoning should be sequenced or contextualized for vocational application (Kaiser, 2017). TVET geometry curricula tend to retain the structure of general mathematics syllabi, with vocational content appended as illustration rather than serving as the organizing logic of instruction (Acheampong et al., 2026; Susilo et al., 2020).

Table 1. Operationalization of Theoretical Constructs in the Analysis

Theoretical Construct	Operational Indicator (Instrument/Data)	Corresponding Result / Section
Horizontal mathematization (RME)	Presence of vocational context prior to formal geometric content; vocational reference counts (**Table 10**)	5.1.2, 5.1.8 – contextualization occurs but rarely precedes formalization
Vertical mathematization (RME)	Spatial visualization, technical drawing, and estimation task frequency (**Table 4**)	5.1.3 – these categories are the least-observed content types (26.7–40%)
Guided reinvention (RME)	Lesson opening sequence: definition-first vs. task-first (Observation Schedule, Item 2)	6.1 – 93.3% of lessons opened with definitional content
Experiential learning cycle (Kolb)	Presence of concrete experience, reflection, and closure activities (**Table 2**; **Figure 1**)	5.1.1, 5.1.9 – closure and reflective consolidation observed in ≤20% of lessons
Zone of proximal development / mediation (Vygotsky)	Facilitator questioning type and frequency (**Table 9**); tool-mediated instruction (**Table 5**)	5.1.7 – vocationally-linked questions average <2 per lesson
Social mediation via collaboration (Vygotsky)	Interview reports of cross-disciplinary facilitator/instructor interaction	5.2.2 – structured collaboration reported as an exceptional enabling condition

Freudenthal (1991) critiqued the "anti-didactical inversion," the presentation of mathematics as a finished product rather than an activity of organizing experience. As Borba et al. (2017) argue, the challenge is not to add vocational examples but to restructure the relationship between mathematical and vocational knowledge.

Factors Influencing Abstract-Practical Integration

At the facilitator level, subject-matter confidence, vocational knowledge, and willingness to depart from textbook-driven instruction are critical (Boateng & Awuah, 2026; Jones & Tzekaki, 2016), and industry experience strengthens vocational connections (Zhang, 2025). Institutional factors such as measurement tools, workshop access, and timetable pressure also shape integration (Power & Handley, 2017; Buckley et al., 2019). At the curriculum level, specifying vocational application as a learning outcome influences practice (Singh et al., 2024). Assessment arguably exerts the strongest influence: examination demands determine instructional priorities regardless of curriculum policy (Jones, 2006), so facilitators teach to procedural-recall examinations (Anamuah-Mensah & Asabere-Ameyaw, 2011). Perceived irrelevance is among the strongest learner-level predictors of disengagement (Acheampong & Awuah, 2025; Akansiaba et al., 2025; Madrilejos, 2024). Overall, the literature offers more diagnosis than solution, with few detailed classroom-level accounts of how integration is or is not enacted.

METHODS

Research Design

This study adopts a qualitative, multi-method design suited to generating a grounded, triangulated account of current instructional practice and stakeholder perspectives on the conditions shaping it (Lakin et al., 2024; Pantić, 2017; Turner & Meyer, 2000). Data are drawn from structured classroom observations and semi-structured interviews, triangulated across data sources and across mathematics facilitators, vocational instructors, learners, and administrators to strengthen credibility.

Participants and Sampling

A total of 11 participants took part: three mathematics facilitators (MF-01–MF-03), three vocational instructors in carpentry, welding, and electrical trades (VI-01–VI-03), two administrators (AD-01–AD-02), and three TVET learners (L-01–L-03). Purposive sampling ensured direct, relevant experience with geometry instruction or curriculum design, yielding information-rich cases for nuanced perspectives on geometry pedagogy (Abylkassymova et al., 2022; Ntereke & Mphunyane, 2025; Sunzuma & Maharaj, 2019).

Participant Inclusion Criteria and Selection Rationale

Participants were selected based on the following criteria:

Mathematics Facilitators (n=3): Currently teaching Core Mathematics at the pre-tertiary TVET level, with a minimum of three years of teaching experience, and responsible for geometry instruction. Facilitators were recruited from different institutions to capture variation in institutional contexts.

Vocational Instructors (n=3): Currently teaching carpentry, welding, or electrical installation, with a minimum of five years of trade instruction experience, and able to speak to the geometric demands of their trade. Instructors were selected from the same institutions as the mathematics facilitators to enable discussion of cross-disciplinary collaboration.

Administrators (n=2): Holding positions as principal, vice-principal, or academic coordinator, with direct oversight of the mathematics curriculum and/or TVET programs, and a minimum of three years in administrative roles.

Learners (n=3): Enrolled in TVET programs at participating institutions, having completed at least one year of study (to ensure familiarity with geometry instruction), and representing different trade specializations.

Institutions were selected purposively based on three criteria: (1) offering pre-tertiary TVET programs with Core Mathematics as a compulsory subject; (2) having at least three trade specializations to enable comparison; and (3) accessibility for the research team. Three institutions were selected to provide variation in institutional size, location (urban vs. peri-urban), and resource availability. Participants were identified as "information-rich cases" through consultation with institutional administrators, who recommended facilitators and instructors with diverse experience levels and instructional approaches. This approach ensured that the sample captured variation in teaching experience, trade specialization, and institutional context, enhancing the transferability of findings (Lincoln & Guba, 1985).

Participant Characteristics

Sample Composition and Data Saturation

Table 2 presents the dynamics of the participants of the study. Saturation was assessed separately for the observation and interview datasets rather than assumed a priori. For classroom observations, the researcher tracked new codes across the fifteen sessions; no new descriptive codes were added after the twelfth lesson, and the final three confirmed rather than extended the coding structure. For interviews, no new themes emerged within the facilitator (n=3) or instructor (n=3) subgroups after the fifth and sixth interviews, indicating saturation within those groups. Saturation is not claimed to the same degree for the administrator (n=2) and learner (n=3) subgroups; this asymmetry is treated as a limitation (Section 6.5) rather than resolved. Findings attributed to "administrators" should be read as illustrative of institutional-level perspective rather than saturated across the role; similarly, the three learners cannot represent the full range of specializations or year groups sampled. Learner perspectives on vocational integration, central to Research Question 2, are the least saturated dimension and are interpreted accordingly in Section 6.

Data Collection Procedure

Data collection was conducted over eight weeks across three pre-tertiary TVET institutions in Ghana, following a sequential, multi-method design: classroom observations first, followed by semi-structured interviews.

Table 2. Participant Characteristics

Participant Code	Role	Trade Area	Years of Experience	Institution
MF-01	Mathematics Facilitator	N/A	8	Institution A
MF-02	Mathematics Facilitator	N/A	12	Institution B
MF-03	Mathematics Facilitator	N/A	4	Institution C
VI-01	Vocational Instructor	Carpentry	15	Institution A
VI-02	Vocational Instructor	Welding	9	Institution B
VI-03	Vocational Instructor	Electrical	6	Institution C
AD-01	Administrator	N/A	10	Institution B
AD-02	Administrator	N/A	7	Institution A
L-01	Learner	Carpentry	Year 2	Institution A
L-02	Learner	Welding	Year 3	Institution B
L-03	Learner	Electrical	Year 1	Institution C

Note. Years of experience for learners indicate years of study. N/A indicates role does not involve a trade specialization.

Classroom Observations

Each of the three mathematics facilitators was observed teaching five geometry lessons, yielding fifteen classroom observations, scheduled in consultation with facilitators to avoid disrupting normal routines. Lessons covered geometry topics typical of the pre-tertiary TVET Core Mathematics curriculum, including angles, area and perimeter, the Pythagorean theorem, and geometric constructions. The first author conducted all observations using the structured Classroom Observation Schedule (Instrument 1; Section 4.4.1), which guided documentation across ten dimensions, combining structured checklist items with open notes to capture contextual detail. Each session lasted the full regular class period, 45 to 60 minutes. To minimize the Hawthorne effect, the researcher spent an initial acclimatization period in each classroom before formal observation began. All observations were non-participatory; the researcher sat at the back of the classroom without interacting with facilitators or learners.

Semi-Structured Interviews

Following the observations, semi-structured interviews were conducted with all eleven participants across the four role groups described in Section 4.2. Interviews were scheduled at times and locations convenient for participants, typically in private offices or quiet classrooms after instructional hours, and lasted 30 to 60 minutes, conducted by the first author using the Semi-Structured Interview Guide (Instrument 2; Section 4.4.2), with probes eliciting elaboration while preserving comparability across participants. All interviews were conducted in English, the language of instruction in Ghanaian TVET institutions, and audio-recorded with informed consent; in one instance, recording was paused at a participant's request and resumed with permission. Field notes captured non-verbal cues, contextual details, and initial analytical observations.

Data Management and Triangulation

All observation notes and interview recordings were transcribed verbatim within 48 hours and anonymized using participant codes. Transcripts were stored securely with password protection, accessible only to the research team. Triangulation was facilitated by temporal sequencing: observations provided a direct account of enacted practice (Research Question 1), while subsequent interviews let participants reflect on and elaborate the conditions documented observationally (Research Question 2), and let the researcher follow up on specific observed practices with the facilitators concerned.

Instrument Development

Data collection was supported by two purpose-built instruments, each addressing one research question and drawing on a designated participant group. Both were developed to be coherent with the study's theoretical framework and design, providing complementary data allowing findings to be triangulated across methods and roles (Basetto & Galli, 2020; Firdaus et al., 2025; Xiong, 2024).

Instrument 1: Classroom Observation Schedule

Instrument 1 is a structured classroom observation schedule addressing Research Question 1: how geometry concepts are currently taught. It guides observers through ten dimensions: (1) lesson structure, (2) vocational integration, (3) content presented, (4) representation modes, (5) learner engagement, (6) assessment practices, (7) facilitator-learner interaction, (8) contextualization, (9) lesson closure, and (10) resources and materials. It combines structured checklists with open notes capturing contextual detail a checklist alone cannot, plus a final section on the strength of the abstract-practical connection evident. It was completed during or immediately after each lesson by the trained researcher.

Instrument 2: Semi-Structured Interview Guide (Research Question 2)

Instrument 2 is a semi-structured interview guide addressing Research Question 2: the factors facilitating or hindering the connection between abstract geometry and practical vocational application. It is organized into four role-specific question sets, for facilitators, instructors, learners, and administrators, each covering current practice, facilitating factors, hindering factors, and recommendations, with probes eliciting elaboration while preserving comparability. Role-specific tailoring ensures relevance: facilitators are asked about their own practices; instructors about geometric knowledge demands and collaboration; learners about their experience and its perceived relevance; and administrators about institutional resources and policy.

Trustworthiness

To ensure trustworthiness, Lincoln and Guba's (1985) four criteria, credibility, transferability, dependability, and confirmability, were systematically operationalized. Credibility was enhanced through triangulation across observation schedules and interview transcripts, allowing cross-verification across participant groups and methods (Moustakas, 1994). Member checking returned preliminary thematic summaries to selected participants (two facilitators, two instructors, two learners) for validation. Dependability was established through systematic double-coding: two researchers independently coded a 30% subset of interview transcripts and observation notes, resolving discrepancies through discussion. The final codebook was applied consistently, yielding a strong inter-coder Kappa of 0.84 (Miles et al., 2014). Confirmability was supported by a comprehensive audit trail documenting methodological decisions, coding, theme development, and reflexive notes, including raw data, data reduction products, and reconstruction products, enabling external reviewers to trace the analytic process (Shenton, 2004). Finally, transferability was addressed through thick descriptions of institutional profiles, classroom settings, participant characteristics, and instructional conditions observed, enabling readers to assess applicability to similar pre-tertiary TVET settings elsewhere (Lincoln & Guba, 1985). Collectively, these practices strengthened the study's methodological rigour.

Positionality of the Researchers

The research team occupies a dual positionality, both insider and observer, within Ghanaian TVET mathematics education. The first author, a mathematics educator with experience in pre-tertiary TVET instruction, brings deep understanding of the institutional environment and curriculum expectations. The second author, an established researcher with extensive experience in curriculum analysis and facilitator development in Ghana, adds scholarly perspective on the structural and policy dimensions of the divide. This professional proximity offered advantages: it facilitated trust and rapport across the four stakeholder groups, who were more willing to share candid reflections knowing the researchers had an informed appreciation of their professional realities, and the team's familiarity with Ghana's TVET curriculum, examination system, and institutional hierarchies enriched interpretation of how policy intentions translate into classroom practice.

The insider position also posed risks of bias in interpreting classroom practices that may reflect systemic shortcomings within institutions the researchers are professionally connected to. Reflexive practices were employed throughout, including a research journal and peer debriefing with external colleagues, and triangulation and member checking served as further checks against overreliance on any single interpretive frame. Two concrete steps also managed insider bias: the first author did not observe or interview in institutions where he held an ongoing supervisory relationship, and the independent double-coding of a 30% subsample (Section 4.5) served specifically as a check on insider bias. Two further risks warrant acknowledgment. First, the Hawthorne effect cannot be ruled out given the small number of lessons per facilitator (five); if facilitators modified practice toward greater integration when observed, the low integration levels reported in Section 5.1.2 may understate rather than overstate typical practice, carried forward as a limitation in Section 6.5. Second,

the researchers' shared commitment to strengthening TVET mathematics education could predispose analysis toward systemic rather than facilitator-level explanations; this is addressed, though not eliminated, by the divergent readings in Section 6.1 and by member checking. The researchers' academic positioning within Ghanaian higher education, combining scholarly rigour with practical relevance, shaped the study's theoretical framing and interpretive orientation, generating findings that are empirically grounded and actionable for curriculum development and instructional reform.

Ethical Considerations

The study was conducted in accordance with standard ethical research protocols, including institutional ethics approval, informed consent, voluntary participation, and confidentiality and anonymity in reporting classroom observation, interview, and survey data.

RESULTS

Research Question 1: How Are Geometry Concepts Currently Taught in Pre-Tertiary TVET Institutions?

Fifteen classroom observations were conducted across participating institutions, covering geometry lessons in a range of trade specializations. Findings are organized around ten thematic dimensions – lesson structure, vocational integration, content, representation, engagement, assessment, interaction, contextualization, closure, and resources – depicting how geometry instruction is currently enacted. A note on the percentages reported throughout this section: they describe patterns within the fifteen purposively sampled lessons and are descriptive proportions of that sample, not population estimates. No confidence intervals or significance tests are implied; “80%” means 12 of 15 observed lessons, not an inferential claim about TVET geometry instruction broadly.

Lesson Structure and the Dominance of Facilitator-Centered Instruction

The most striking structural feature of geometry lessons was the extent to which instruction was organized around the facilitator rather than learner activity. Lecture and direct explanation were the primary method in 12 of 15 lessons (80%), averaging 25 minutes per session (**Table 3**). Individual seatwork was the second most common (60%), and question-and-answer exchanges occurred in 67% of lessons, though predominantly factual recall (Section 5.1.7).

Table 3. Distribution of Primary Teaching Methods Observed

Teaching Method	Frequency of Use	Duration (Average)	Key Observations
Lecture/Explanation	12/15 (80%)	25 min	Facilitator-dominated; limited learner interaction
Demonstration	8/15 (53%)	10 min	Often used for tool use or procedures
Question-and-Answer	10/15 (67%)	5 min	Mostly low-level recall questions
Group Work	3/15 (20%)	20 min	Limited; often facilitator-facilitated
Practical/Hands-on Activity	6/15 (40%)	20 min	Primarily in workshop-based sessions
Use of Manipulatives/Models	4/15 (27%)	5 min	Basic geometric models
Individual Seatwork	9/15 (60%)	10 min	Textbook problems and worksheets
Problem-Solving Exercises	7/15 (47%)	15 min	Mostly routine problems

Note. Multiple methods could be observed within a single lesson. Percentages reflect the proportion of the 15 observed lessons in which each method was used. These are descriptive proportions of the observed sample, not inferential statistics or population estimates.

Active and collaborative modes were substantially less common. Practical or hands-on activities occurred in only 40% of lessons, largely confined to workshop-adjacent sessions, and group work in just 20%, typically facilitator-directed. Demonstration (53%) mainly modeled tool use or procedure rather than geometric principles, and manipulatives were used only briefly (27%, averaging five minutes).

This pattern reflects a predominantly transmission-oriented approach in which facilitators present, demonstrate, and assign exercises while learners remain largely receptive. As one facilitator acknowledged:

> *"The syllabus is so packed that I have to cover the definitions and formulas first. There is no time to let learners discover things on their own." – (MF-01)*

This transmission orientation is consistent with Freudenthal's (1991) critique of ready-made mathematics transmission; geometric content is delivered rather than reinvented from experience: this is the guided-reinvention construct in **Table 1**, operationalized here through the low incidence – a characteristic any bridging framework must address.

Vocational Integration in Geometry Instruction

A second key finding concerns the extent to which geometry instruction connected to learners' vocational specializations (**Table 4**). The overwhelming majority of lessons demonstrated limited or no meaningful integration: 13.3% (n=2) were purely abstract, and the largest category, minimal integration (33.3%), consisted of brief, incidental trade-relevance mentions. Moderate integration accounted for a further 26.7%; only 20% reached substantial integration, and full integration was observed in a single lesson (6.7%).

Taken together, these figures indicate that 73.3% of observed lessons demonstrated either no integration or only superficial connections between geometry and vocational practice, at odds with the dual mandate TVET geometry instruction is expected to serve. One facilitator acknowledged that "carpenters need to understand angles" but returned immediately to abstract calculation (Observation 4); by contrast, the exceptional cases of substantial or full integration stood out, as when learners measured actual metal pieces for a practical project (Observation 12).

> *"The facilitator mentioned that carpenters need to understand angles but continued with abstract calculations without further connection." (Observation Notes, Observation 4)*

> *"Learners were measuring actual metal pieces and calculating areas required for a practical project." (Observation Notes, Observation 12)*

Interviews with facilitators helped explain why this integration was so rare. MF-02, who had prior engineering experience and achieved higher integration scores, explained:

Table 4. Extent of Practical Vocational Integration Observed Across Lessons

Level of Integration	Frequency (n=15)	%	Characteristic Features
No integration observed	2	13.3%	Purely abstract geometry with no trade connections
Minimal integration	5	33.3%	Brief mention of trade examples (e.g., "carpenters use angles")
Moderate integration	4	26.7%	Vocational examples discussed but not applied
Substantial integration	3	20.0%	Learners engage with vocational tasks applying geometry
Full integration	1	6.7%	Geometry taught through authentic vocational activities

Note. Integration levels reflect the researcher's holistic rating of each lesson using the observation schedule categories. As above, these figures are descriptive proportions of the 15 observed lessons and are not intended as inferential or population-level estimates.

> *"When you have worked in industry, you can see the connections naturally. But most of us went straight from university to teaching. We know the theory but not the practical application." – (MF-02)*

The vocational references that do occur function as what Freudenthal (1991) termed horizontal mathematization—translating practical contexts into mathematical language—but rarely proceed to the vertical mathematization that would formalize and generalize that content geometrically. This reflects the structural insulation between academic and vocational knowledge documented in the pedagogic-codes literature (Bernstein, 2000).

Nature of Geometric Content Presented

The geometric content presented reflected a predominantly formal, definition-and-formula orientation. Definitions were present in every lesson (100%), and formulas and calculations featured in 93.3% (n=14), the most consistently present content types (**Table 5**). Measurement techniques appeared in 80% of lessons, mostly through instrument use, and theorems and proofs in 53.3%. Content types more directly supporting abstract-practical bridging were substantially less common: spatial visualization exercises in only 40% of lessons, technical drawing or blueprint reading, among the most trade-relevant skills, in just 33.3%, and practical estimation tasks in only 26.7%. The formal, propositional dimensions of geometry thus dominate coverage while applied, spatial dimensions remain marginal.

The near-absence of technical drawing is particularly striking given its centrality to vocational practice. As VI-01 (Carpentry Instructor) explained:

> *"In carpentry, we work from drawings every day. But when I ask learners about the geometry they learned, they say they never saw a blueprint in math class." – (VI-01)*

Representation Modes Employed in Geometry Instruction

The representational landscape of observed lessons was dominated by visual-diagrammatic and symbolic modes, with limited use of concrete or vocational representations (**Table 6**). Whiteboard or textbook diagrams were the primary mode in 93.3% of lessons, and formal symbolic notation appeared in 86.7%. Tool-based representations, rulers, protractors, and compasses, were observed in 66.7%. However, representations engaging learners with the materiality of vocational work were far less common. Concrete objects or 3D models were present in 40% of lessons, vocational materials in only 26.7%, and digital representations in only two lessons (calculator use). This hierarchy, privileging abstract symbolic and diagrammatic modes over concrete and vocational ones, is consistent with the broader instructional pattern.

Table 5. Types of Geometric Content Observed in Lessons

Content Type	Frequency (n=15)	Example Topics Observed
Definitions of geometric terms	15 (100%)	Types of angles, triangles, circles
Formulas and calculations	14 (93.3%)	Area, perimeter, volume calculations
Measurement techniques	12 (80%)	Using rulers, protractors, measuring tapes
Theorems and proofs	8 (53.3%)	Pythagorean theorem, angle properties
Spatial visualization exercises	6 (40%)	Mental rotation, 2D to 3D interpretation
Technical drawing/Blueprints	5 (33.3%)	Reading blueprints, creating simple drawings
Practical estimation tasks	4 (26.7%)	Estimating measurements before verifying

Note. Multiple content types could be present within a single lesson. Frequencies reflect the number of lessons in which each content type was observed.

Table 6. Representation Modes Employed in Geometry Lessons

Representation Mode	Frequency (n=15)	Notes on Usage
Visual/Diagrammatic	14 (93.3%)	Primary mode; whiteboard drawings, textbook diagrams
Symbolic/Formal notation	13 (86.7%)	Formulae, geometric notation, equations
Tool-based	10 (66.7%)	Rulers, protractors, compasses
Concrete objects/Models	6 (40%)	3D shapes, models of geometric figures
Vocational materials/workshop items	4 (26.7%)	Building materials, metal pieces, wood samples
Digital/Technology-based	2 (13.3%)	Limited; occasional use of calculators

Note. Multiple representation modes could be observed within a single lesson.

Table 7. Learner Participation Levels Observed Across Lessons

Participation Level	Frequency (n=15)	Description
High (most actively engaged)	3 (20%)	Learners actively answering, discussing, asking questions
Moderate (some engaged)	6 (40%)	Core group participates; others passive
Low (few engaged)	4 (26.7%)	Limited verbal participation; mostly listening
Minimal (learners passive)	2 (13.3%)	Learners largely silent; few responses

Note. Participation level reflects the researcher's holistic rating based on observed learner behavior throughout the lesson.

Learner Engagement and Evidence of Geometric Reasoning

Learner engagement was predominantly moderate to low (**Table 7**). Only 20% of lessons showed high engagement, with most learners actively contributing; 40% showed moderate engagement, where a core group participated while others remained passive; and the remaining 40% showed low or minimal engagement with largely passive listening. Evidence of spatial and geometric reasoning beyond procedural execution was similarly limited (**Table 8**). The most frequent indicator was correct use of geometric vocabulary (53.3%), followed by application of geometric principles (46.7%) and interpretation of drawings (40%). Active measurement was observed in 33.3%, and conceptual explanation—learners articulating why a principle holds rather than merely executing it—in only 26.7%; in 20% no explicit reasoning was observed.

Interview data provided insight into why engagement was so limited. L-01 (Carpentry Learner) explained:

> *"The math class feels like a different world. I don't see how that relates to the wood I'm cutting." – (L-01)*

This perceived irrelevance, as Madrilejos (2024) found, is a strong predictor of disengagement. The scarcity of facilitator questioning that would move learners toward assisted conceptual articulation (Section 5.1.7) is the mechanism sustaining this limited development.

Table 8. Evidence of Spatial and Geometric Reasoning Observed

Evidence Type	Observed (n=15)	Percentage	Example
Learners applying geometric principles to tasks	7	46.7%	Using formulas to solve practical problems
Learners interpreting drawings/diagrams	6	40.0%	Reading technical drawings
Learners making measurements	5	33.3%	Measuring angles/lengths
Learners using geometric vocabulary	8	53.3%	Correct terminology used in responses
Learners explaining geometric concepts	4	26.7%	Explaining why formulas work
No explicit reasoning observed	3	20.0%	Rote memorization of procedures

Note. Multiple evidence types could be observed within a single lesson

Assessment and Feedback Practices

Assessment practices were heavily weighted toward oral questioning and written procedural exercises. Oral questioning, used in 93.3% of lessons, was the primary mechanism for checking understanding, and written exercises featured in 80% (**Table 9**). Informal facilitator observation during tasks was recorded in 53.3%. Assessment of applied competence was far less common: homework occurred in 40% and practical demonstration in only 33.3%, mainly in workshop-linked sessions. The nature of assessment tasks reflected the broader instructional pattern: predominantly procedural recall rather than problem-solving or practical demonstration, likely reinforcing geometry's abstract character since assessment demands shape learning activities. A bridging framework must therefore address assessment design alongside instruction, embedding vocational tasks as legitimate, regular modes of assessing geometric competence.

Facilitator-Learner Interaction and Questioning Patterns

Facilitator questioning patterns revealed a strong bias toward low-level factual recall (**Table 10**): 12 to 18 low-level questions per lesson (e.g., “What is the formula for area?”) against only 3 to 6 higher-level questions. Questions promoting geometric reasoning averaged 2 to 5 per lesson, and questions linking geometry to vocational practice averaged fewer than two, absent entirely from several observations.

This questioning profile is consequential for the development of mathematical reasoning. Field notes recorded that facilitator questioning predominantly focused on procedural recall, with learners rarely asked to justify their reasoning or connect to vocational contexts (Field Notes, Observation 8).

Interviews revealed why this pattern persists. MF-01 explained:

> *"I ask basic questions to check if they understand the formulas. When I ask harder questions, many learners just don't answer. So I keep it simple." – (MF-01)*

This reflects a rational adaptation to time pressure and learner preparedness, but from a Vygotskian perspective, it represents a missed mediational opportunity: it is through questioning that learners are guided from concrete task performance toward reflective articulation of the geometric principles involved (Vygotsky, 1978).

Table 9. Assessment Methods Observed Across Lessons

Assessment Method	Frequency (n=15)	Purpose/Context
Oral questioning	14 (93.3%)	Checking understanding and factual recall
Written exercises	12 (80.0%)	Textbook problems and worksheets
Facilitator observation	8 (53.3%)	Informal monitoring during tasks
Practical demonstration	5 (33.3%)	Hands-on assessment in workshops
Homework assignment	6 (40.0%)	Extension of classwork

Note. Multiple assessment methods could be observed within a single lesson

Table 10. Facilitator Questioning Patterns Observed

Question Type	Freq. (Avg per Lesson)	Example Questions
Low-level (factual recall)	16	What is the formula for area?
Higher-level (explanation/analysis)	6	Why does this formula work?
Open-ended questions	3	How would you solve this problem?
Questions requiring geometric reasoning	5	What do these angles tell us?
Questions linking to vocational practice	2	How would this apply to your trade?

Note. Frequencies represent the range observed across the 15 lessons. Low-level questions were the dominant type in all observed sessions

Contextualization and Vocational References

A more granular analysis of vocational references reinforces the integration findings above (**Table 11**). While 53.3% of lessons mentioned specific trades by name and 40% used vocational terminology (e.g., "plumb," "level," "square"), these rarely functioned as the organizing logic of instruction. References to workshop procedures (33.3%) and workplace applications (26.7%) typically remained illustrative, trade examples appended to an abstract sequence rather than the starting point for reinventing geometric understanding. This pattern matches what the pedagogic-codes literature terms illustrative contextualization: vocational examples added to an essentially academic pedagogical logic without restructuring it. Observed contextualization largely remained at Freudenthal's horizontal-mathematization level, translating practical contexts into mathematical language, without the vertical mathematization that would formalize and generalize deeper geometric understanding.

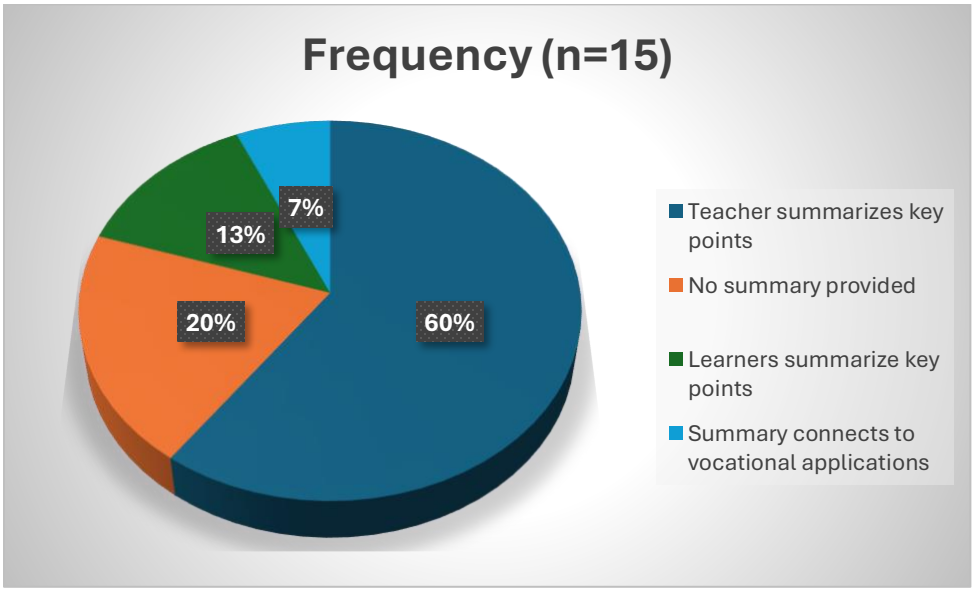
Lesson Closure Practices

Lesson closure, with particular potential for reinforcing abstract-practical connections, was generally underdeveloped (**Figure 1**). Facilitator-led summaries were provided in 60% of lessons, but in 20% none was provided. Learner-led summaries occurred in only 13.3%, forfeiting the reflective consolidation Kolb's cycle identifies as essential. Closure explicitly connecting content to vocational application was observed in only a single lesson (6.7%).

Table 11. Explicit Vocational References Observed Across Lessons

Reference Type	Frequency (n=15)	Illustrative Examples
None	2 (13.3%)	No reference to trade context throughout the lesson
Mention of specific trades	8 (53.3%)	References carpenters, electricians, and mechanics
Reference to workshop procedures	5 (33.3%)	Measuring, cutting, and constructing referenced
Use of vocational terminology	6 (40.0%)	"Plumb", "level", "square" used in explanations
Discussion of workplace applications	4 (26.7%)	Real-world problem scenarios discussed

Note. A single lesson could include multiple reference types. Two lessons (13.3%) contained no vocational references of any kind



Note. Percentages reflect the proportion of the 15 observed lessons.

Figure 1. Lesson Summary Patterns (Source: The authora)

As MF-03 reflected:

> *"I usually just go over the main points at the end. I hadn't thought about connecting it to their trades during the closure—the summary is more about making sure they remember the formula." – (MF-03)*

This absence of reflective consolidation, particularly of vocational connections, represents a missed opportunity to strengthen the abstract-practical bridge Kolb's (1984) experiential learning cycle identifies as essential.

Resources and Materials Used

The resource environment showed a sharp gradient from high-frequency use of conventional classroom tools to very limited use of vocational and digital resources (**Table 12**). Chalkboards were used in all 15 lessons, textbooks in 86.7%, and geometric instruments in 80%, suggesting tool-based measurement is a consistent feature of instruction. Beyond these conventional resources, the pattern drops sharply.

Workshop tools appeared in only 40% of lessons, concentrated near workshop spaces. Physical models were present in 26.7%, and technical drawings or blueprints, arguably the most trade-relevant visual resource, in only 20%. Digital or ICT resources were observed in just two lessons. This near-absence both reflects the broader instructional approach and constrains the integration achievable under current conditions.

Summary of Findings for Research Question 1

The observational data converge on a coherent picture of geometry instruction that is predominantly facilitator-centered, formally oriented, and weakly connected to learners' vocational specializations. Instruction centres on lecture, textbook exercises, and procedural question-and-answer exchanges; content is dominated by definitions, formulae, and theorems, with spatial visualization, technical drawing, and estimation appearing only in a minority of lessons.

Representation is overwhelmingly symbolic and diagrammatic; engagement is moderate to low; assessment favours oral recall and written exercises over practical demonstration; vocational references are largely superficial; closure rarely consolidates abstract-practical connections; and the resource environment, adequate conventionally, is limited in vocational materials, technical drawings, and digital tools.

These findings document the empirical baseline from which the conceptual framework developed in this study departs. They confirm that the abstract-practical divide identified in the literature is a demonstrable feature of current geometry instruction in Ghanaian pre-tertiary TVET, and specify the dimensions along which a bridging framework must intervene.

Table 12. Resources and Materials Observed Across Geometry Lessons

Resource Type	Observed (n=15)	Percentage	How Used
Chalkboard/Whiteboard	15	100%	Diagrams, formulas, and worked-examples
Textbooks/Worksheets	13	86.7%	Problem sets, and reference examples
Geometric instruments	12	80%	Measurement and drawing construction
Workshop tools/equipment	6	40%	Practical measurement activities
Models/Manipulatives	4	26.7%	Demonstrating 3D shapes
Technical drawings/Blueprints	3	20%	Interpreting geometric components of drawings
Digital/ICT resources	2	13.3%	Calculators, occasional instructional videos

Note. Multiple resource types could be observed within a single lesson

Research Question 2: What Factors Facilitate or Hinder the Connection Between Abstract Geometry Concepts and Practical Vocational Applications in Pre-Tertiary TVET?

Data addressing Research Question 2 were generated through semi-structured interviews with the eleven participants across the four role groups described in Section 4.2. Thematic analysis produced five themes: current instructional disconnects, facilitators, hindering factors, cross-cutting patterns, and framework recommendations, presented below with selected excerpts.

Current Instructional Practice and the Theory-Practice Disconnect

Whereas Section 5.1.2 documented the extent of vocational integration observed directly in the classroom, interview data reveal how this disconnect is experienced across stakeholder groups. Across all groups, a persistent structural disconnect between mathematics classroom instruction and vocational workshop practice was the most consistently identified feature of current teaching. MF-01 (8 years) described a transmission-oriented approach driven by examination scheduling:

> *"The syllabus is so packed that I have to cover the definitions and formulas first. There is no time to let learners discover things on their own. The exam only tests formulas anyway." – (MF-01)*

MF-03 (4 years) had attempted an early collaboration with the carpentry instructor on rafter-length calculations, reported as highly engaging but discontinued due to timetabling difficulties:

> *"We tried it once—the learners were so engaged! But then the timetable changed and we never found time to coordinate again." – (MF-03)*

Vocational instructors independently corroborated the disconnect from the workshop side. VI-01 (Carpentry, 15 years) found learners could not transfer geometric knowledge to practical tasks, requiring him to reteach principles himself despite limited pedagogical expertise, at one point discovering mid-lesson that he had conflated slope ratio with angle when teaching roof pitch:

> *"Geometry is the foundation of carpentry. But here's the problem—learners come to my workshop and they can't apply what they learned in mathematics class." – VI-01 (Carpentry Instructor)*

> *"The mathematics class feels like a different world. I don't see how that relates to the wood I'm cutting." – L-01 (Carpentry Learner, Year 2)*

Facilitators of Abstract-Practical Connection

Thematic analysis identified facilitating factors operating at three levels: personal, institutional, and curricular. At the personal level, facilitators with prior industry or trade experience showed a markedly stronger capacity to make vocational connections. MF-02 (12 years, with prior engineering experience) drew routinely on her industry background:

> *"When I explain angles, I talk about how they used the same principle when I worked on construction sites. I can see them paying more attention when I make it real." – (MF-02)*

L-02 (Welding, Year 3) described how relating angle bisection to joining pipes made a principle comprehensible where abstract instruction had not:

> *"When the teacher said this is about joining pipes at the right angle, it just clicked. Before that, I didn't see why we were learning it." – (L-02)*

At the institutional level, the most powerful facilitators were structured opportunities for cross-disciplinary interaction. AD-01 described "integration weeks," during which mathematics facilitators visited the workshop and vocational instructors attended mathematics classes, supplemented by quarterly alignment meetings:

> *"When we bring them together, they realize how much they could help each other. The math teacher sees what the students actually need in the workshop." – (AD-01)*

VI-02 (Welding, 9 years) offered the clearest example: a mathematics facilitator who observed his welding sessions restructured her lessons around the joint types learners were practising:

> *"She came to my workshop and watched how I teach the students to weld at different angles. Then she changed her math lessons to focus on those angles. The students said, 'Now we understand why we're learning this!' When facilitators collaborate, learners understand." – VI-02 (Welding Instructor)*

At the curricular level, participants noted the revised TVET syllabus had begun including suggested practical applications, used by some facilitators as a prompt for contextualized instruction. However, MF-01 noted these mentions (e.g., "learners should be able to apply area to tiling") provided no pedagogical guidance or worked examples:

> *"The syllabus mentions practical applications but doesn't tell us how to teach them. I'm left to figure it out myself." – (MF-01)*

Hindering Factors

Five categories of hindering factor emerged: facilitator-related challenges, learner-related challenges, institutional constraints, resource limitations, and assessment barriers. These were consistent across participant groups and mutually reinforcing rather than discrete.

Facilitator-related challenges centred on limited vocational knowledge and time pressure. MF-03 described a learner from a plumbing background correcting an unrealistic example, generating embarrassment that deterred further contextualization attempts:

> *"I tried to use plumbing as an example once, and a student said, 'That's not how it works in real life.' I was embarrassed and haven't tried again." – (MF-03)*

This illustrates how absent trade-specific knowledge can discourage even motivated facilitators. MF-02 identified time pressure as the second major barrier: 15 geometry topics compete for 40 timetabled periods per term:

> *"If I try to integrate vocational examples properly, I fall behind the syllabus. The exam comes regardless of whether I've covered everything." – (MF-02)*

Learner-related challenges included weak foundational knowledge and classroom anxiety. VI-03 (Electrical, 6 years) regularly encountered learners unable to distinguish area from perimeter, requiring workshop time to be reallocated to remediation:

> *"I have to spend workshop time teaching basic measurement because they didn't learn it properly in math. It wastes time I should be spending on electrical skills." – (VI-03)*

L-03 (Electrical, Year 1) described suppressing comprehension difficulties to avoid embarrassment:

> *"Sometimes I don't understand, but I'm afraid to ask. The other students will think I'm stupid." – (L-03)*

Institutional constraints included staffing, timetabling, and resource availability. AD-01 identified a shortage of "hybrid facilitators" able to work across mathematical and vocational domains, compounded by non-overlapping class schedules:

> *"We don't have enough teachers who understand both math and trades. And even when we do, their timetables don't allow for joint planning." – (AD-01)*

VI-01 noted that reliance on scrap materials prevented learners from developing the measurement precision that accurate geometric work demands:

> *"We work with scrap wood—nothing is straight. How can they learn precision when the materials are imprecise?" – (VI-01)*

Assessment system barriers were identified by all groups as the deepest structural constraint. MF-02 and AD-02 both articulated the core dynamic: as long as national examinations test only abstract, procedural knowledge, facilitators will teach to that examination regardless of any framework or initiative:

> *"If the exam asked them to apply geometry to a real situation, I would teach that way. But the exam drives the teaching." – MF-02*

AD-02 elaborated on the assessment challenge:

> *"We've tried to encourage more practical teaching, but the examination board hasn't changed. Until they do, teachers will always prioritize what is tested." – (AD-02)*

This assessment misalignment creates a rational response at the facilitator level: teaching to the examination is time-efficient and aligns with accountability metrics, even when it undermines vocational integration.

Cross-Cutting Theme Analysis

Table 13 presents the consistency of themes across participant groups, revealing which factors are experienced most uniformly across stakeholder perspectives and which show variation.

Table 13. Cross-Cutting Theme Analysis: Consistency of Facilitators and Barriers Across Participant Groups

Theme	Mathematics Facilitators	Vocational Instructors	Learners	Administrators	Consensus Level
Theory–practice disconnect	Struggle to create relevance	Frustrated by lack of transfer	Confused by abstraction	Aware of the issue	High
Need for collaboration	Value it; rarely practised	Desire more joint planning	Want to see connections	Recognize need; lack structures	High
Assessment as barrier	Facilitate to the exam	Assessment misses application	Worry about exam questions	Advocate reform; change is slow	High
Resource limitations	Need visual materials	Need measuring tools	Want hands-on practice	Acknowledge constraints	Medium
Facilitator confidence	Lack vocational knowledge	Lack pedagogical knowledge	Affected by facilitator clarity	Need for professional development	High
Curriculum flexibility	Rigid syllabus pressures	Limited time for theory	Want practical relevance	Balancing priorities	Medium

Note. Consensus level reflects the degree to which the theme was raised spontaneously and consistently across all participant groups in the interview data.

Summary of Findings for Research Question 2

The interview data converge on a picture in which the theory-practice disconnect documented observationally in Section 5.1 is experienced by all stakeholder groups as structurally embedded rather than an individual pedagogical failing. Enabling factors are present only in exceptional cases, while inhibiting factors- assessment misalignment, workload, limited vocational knowledge, resource constraints, scheduling- are pervasive. A bridging framework must address institutional and assessment conditions, not only instructional design.

DISCUSSION

The Persistence of Transmission-Oriented Geometry Instruction in TVET

That 80% of lessons centered on lecture and direct explanation, with practical activities in only 40% and full vocational integration in one lesson, reflects a structural condition documented across TVET systems. Wedege (2010) argued that school and workplace mathematics are distinct epistemic practices, and Regnier (2014) found that TVET mathematics curricula in several national systems still mirror general education sequences. The present findings show this disconnect enacted lesson by lesson through questioning, representation, and the near-absence of closure. A divergent view holds that well-sequenced direct instruction can produce strong conceptual outcomes (Hiebert et al., 2002) and that procedural instruction may build schemas learners adapt in practice, termed "webbing" (Hoyles et al., 2013). Nevertheless, the evidence here supports the deficit reading. Fewer than two vocational-linked questions per lesson, limited concrete materials, and moderate-to-low engagement suggest the scaffolding that would make "webbing" plausible was absent. Moreover, 93.3% of lessons opened with definitional content while only 6.7% embedded an authentic vocational task, the anti-didactical inversion Freudenthal (1991) criticized. In Ghana, geometry is underrepresented in exit examinations (Folson & Awuah, 2025) and teacher preparation emphasizes content mastery over contextualized pedagogy (Dalby & Noyes, 2016), so transmission-oriented instruction is a rational response rather than an individual failing.

Facilitating and Hindering Factors: Structural Conditions, Not Individual Failings

That enabling factors are exceptional while inhibiting factors are pervasive resonates with prior research. Castles et al. (2018) identified facilitator vocational knowledge as the strongest predictor of contextualized instruction, consistent with MF-02's industry experience, and Kent et al. (2007) described "technomathematical literacies" that pre-service education in Ghana does not systematically develop. The national examination system emerged as the deepest structural barrier. Anamuah-Mensah and Asabere-Ameyaw (2011) and Jones (2006) documented how examinations determine instructional priorities and privilege procedural computation. MF-02's statement, "*if the exam asked them to apply geometry to a real situation, I would teach that way. But the exam drives the teaching*," reflects a rational response to misaligned assessment, not individual conservatism.

Cross-disciplinary collaboration is more contested. VI-02's account of a mathematics facilitator's workshop observations transforming her lesson design suggests collaboration is a powerful enabling condition, consistent with Guile and Griffiths (2001) on "connective" relationships between disciplinary and vocational knowledge. Bernstein (2000), however, notes that strong disciplinary boundaries serve regulative functions, so collaboration needs assessment redesign. MF-03's collaboration with the carpentry instructor lapsed because of timetabling, showing its fragility without institutional support.

Unexpected Findings

Two results were unanticipated. First, only one of fifteen observed lessons achieved full vocational integration, although facilitators endorsed it. Integration appears to require facilitators to absorb design and preparation costs that the timetable and assessment structure (Section 5.2.3) do not accommodate; the single instance (Observation 12) depended on industry background and an ad hoc scheduling window. Integration thus reflects "individual exceptionalism" rather than institutional mechanisms, so professional development should address structural conditions such as collaborative planning time and assessment alignment, not only instructional strategies. Second, the near-total absence of digital representation (13.3%, limited to calculator use) was unexpected given the Ghana Education Service's emphasis on digital literacy. Interviews point to intermittent electricity and absent projection equipment rather than facilitator reluctance. As AD-01 noted:

> *"We have a policy on ICT in education, but the infrastructure isn't there. Most of our classrooms don't have projectors or reliable power." – (AD-01)*

This qualifies the interpretation in Section 6.1: some limitations are infrastructural rather than pedagogical.

Implications for Stakeholders

For facilitators, integration is constrained less by willingness than by time, vocational knowledge, and assessment alignment, so professional development must address collaboration structures and assessment design. For vocational instructors, VI-01 and VI-02 already hold applied geometric knowledge that facilitators lack, so formalizing their role in lesson design and curriculum review is a low-cost step. As VI-02 argued:

> *"If we plan together, both of us can help the students see the connections. The math teacher knows the theory; I know the practice." – (VI-02)*

For administrators, AD-01's account of non-overlapping schedules is corroborated by facilitators and instructors, so dedicated joint planning time or integration weeks could enable integration without additional resources. For curriculum and assessment authorities, all four groups identified examination structure as the deepest barrier (Table 13), so assessment redesign is a precondition for instructional reform, not a parallel track. As AD-02 observed:

> *"We can train teachers, we can provide resources, but if the exam doesn't change, nothing will change." – (AD-02)*

For learners, perceived irrelevance appears to respond to how geometry is taught and assessed rather than being a fixed attitude: engagement increased when L-02 experienced integration (the welding angle example).

LIMITATIONS AND FUTURE RESEARCH

This study has six main limitations. The interview sample was small and uneven, with limited administrator and learner representation, so findings for those groups are indicative rather than saturated. The study covered three purposively selected institutions in one national TVET system, limiting generalizability, and relied on participant reports and researcher contextual knowledge rather than analysis of curriculum or examination documents. The eight-week cross-sectional design could not capture examination-period effects. Researcher positionality may have introduced coding bias, and observer effects may have led integration to be understated rather than overstated. Future research should include longitudinal observation across a full academic term, a larger sample with stronger administrator and learner representation across more trades and year groups, and direct analysis of curriculum and examination documents to test the assessment-architecture explanation in Section 6.2.

CONCLUSION

This study documented how geometry is currently taught in pre-tertiary TVET institutions in Ghana and the conditions shaping how abstract content connects to vocational application. Instruction remains predominantly transmission-oriented, and vocationally integrated teaching is the exception. Enabling conditions occur far less consistently than structural barriers, notably an examination system that rewards procedural recall over applied spatial reasoning. Interpreted through Realistic Mathematics Education, Experiential Learning Theory, and Social Constructivism, the findings show that current sequencing runs counter to each theory's principles: content is rarely reinvented from authentic vocational tasks, lessons seldom move learners through a full experiential cycle, and scaffolding rarely goes beyond procedural correction. These patterns reflect a curriculum, assessment architecture, and institutional environment not designed to support integration, rather than failures of individual facilitators.

For curriculum developers and facilitator educators, strengthening geometry instruction cannot rely on individual initiative alone. Efforts must address assessment design, resource provision, collaboration structures, and professional development, which provide the empirical foundation for the framework developed and validated in a companion study. Findings come from a purposive sample within a single national TVET system and are analytically rather than statistically generalizable. However, the structural conditions documented are widely reported in the regional literature, suggesting the patterns extend beyond the institutions sampled. Future research could use longitudinal observation to capture seasonal variation around examination periods.

Author Contributions: Both authors, DF and FKA, contributed to the conceptualisation, literature search, data collection, analysis, and writing. Both authors have read and approved the final manuscript.

Funding: No funding was received to assist with the preparation of this manuscript.

Declaration of interest: The authors declare no competing or conflicting interests.

Ethical statement: Research did not involve human participants.

AI statement: The authors used the AI tools Paperpal and Claude for language editing and improving fluency. After using these AI tools, we reviewed and verified the final version of our work. We, as the authors, take full responsibility for the content of our published work.

Data sharing statement: No data were collected for this study.

REFERENCES

Abylkassymova, A., Kappasova, S., & Zhadrayeva, L. (2022). The development of mathematical literacy of schoolchildren through discourses. *Bulletin of Abai KazNPU. Series of Physical and Mathematical Sciences, 79*(3), 110–117. https://doi.org/10.51889/9582.2022.49.45.013

Acheampong, P. A., & Awuah, F. K. (2025). The study of mathematics content in the context of vocational education: A systematic review. *Physical and Mathematical Education*, *40*(4), 6–12. https://doi.org/10.31110/fmo2025.v40i4-01

Acheampong, P. A., Awuah, F. K., & Mensah, F. S. (2026). When mathematics remains trade-blind: Recontextualization and workplace mobilization in Ghanaian welding and fabrication education. *Frontiers in Education*, *11, Article 1898213*. https://doi.org/10.3389/feduc.2026.1898213

Akansiaba, C., Boateng, F. O., Appiagyei, E., Adiku, D. K., & Kwakye, D. O. (2025). Investigating perceived student engagement and mathematics achievement in contextualized lessons in TVET schools: Moderating effects, motivation and interest. *European Journal of Contemporary Education and E-Learning*, *3*(4), 46–65. https://doi.org/10.59324/ejceel.2025.3(4).04

Anamuah-Mensah, J., & Asabere-Ameyaw, A. (2011). *Science and mathematics in basic schools in Ghana*. University of Education, Winneba.

Armah, R. B., Cofie, P. O., & Okpoti, C. A. (2017). The geometric thinking levels of pre-service teachers in Ghana. *Higher Education Research*, *2*(3), 98-106. https://www.sciencepublishinggroup.com/article/10.11648/j.her.20170203.14

Asante-Mensa, F.., Asiedu-Addo, S. K., Obeng-Denteh, W., & Armah, G. (2024). The effectiveness of the inquiry-based teaching approach on technical and vocational students' mathematics achievement in Ghana. *International Multidisciplinary Journal of Research and Education*, *2*(1), 70–77. https://doi.org/10.64712/imjre.v2i1.293

Asante-Mensa, F., Asiedu-Addo, S. K., Owusu-Ansah, S., Ansah, I., Agbata, B. C., & Obeng-Denteh, W. (2025). Perceptions of inquiry-based mathematics instruction in technical and vocational schools in Ghana. *Journal of Mathematics Education, 18*(2), 29-46. https://journalofmathed.scholasticahq.com/article/155199-perceptions-of-inquiry-based-mathematics-instruction-in-technical-and-vocational-schools-in-ghana.pdf

Atuhurra, J. & Kaffenberger, M. (2022). Measuring education system coherence: Alignment of curriculum standards, examinations, and teacher instruction in Tanzania and Uganda. *International Journal of Educational Development, 92*, Article 102598, https://doi.org/10.1016/j.ijedudev.2022.102598

Battista, M. T., Winer, M. L., & Frazee, L. M. (2017). How spatial reasoning and numerical reasoning are related in geometric measurement. In E. Galindo & J. Newton (Eds.), *Proceedings of the 39th annual meeting of the North American chapter of the international group for the psychology of mathematics education* (pp. 355-357). Hoosier Association of Mathematics Teacher Educators. https://files.eric.ed.gov/fulltext/ED581320.pdf

Benedičič, U., Duffy, G., & Sorby, S. (2022). An ongoing spatial intervention project in Irish secondary schools for improved engineering education. In *SEFI 50th Annual Conference of the European Society for Engineering Education: Towards a new future in engineering education, new scenarios that European alliances of tech universities open up (pp. 943-951)*. https://doi.org/10.5821/conference-9788412322262.1321

Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, research, critique (Revised ed.)*. Rowman & Littlefield Publishers. https://www.scirp.org/reference/referencespapers?referenceid=2801095

Boateng, R. O., & Awuah, F. K. (2026). Teaching plane geometry through real-life applications: A systematic review. *International Journal of Research and Innovation in Social Science*, *10*(26), 275–284. https://doi.org/10.47772/IJRISS.2026.1026EDU0023

Borba, M. C., Askar, P., Engelbrecht, J., Gadanidis, G., Llinares, S., & Aguilar, M. S. (2017). Digital technology in mathematics education: Research over the last decade. In G. Kaiser (Eds.), *Proceedings of the 13th international congress on mathematical education (pp. 221-233)*. *ICME-13 Monographs*. Springer, Cham. https://doi.org/10.1007/978-3-319-62597-3_14

Buckley, J., Seery, N., & Canty, D. (2019). Investigating the use of spatial reasoning strategies in geometric problem solving. *International Journal of Technology and Design Education, 29*, 341-362. https://doi.org/10.1007/s10798-018-9446-3

Castles, A., Rastle, K., & Nation, K. (2018). Ending the reading wars: Reading acquisition from novice to expert. *Psychological Science in the Public Interest, 19*, 5-51. https://doi.org/10.1177/1529100618772271

Coben, D., & Alkema, A. (2018). Scoping the development of a measure of adults' numeracy (and literacy) practices. In: Safford-Ramus, K., Maaß, J., Süss-Stepancik, E. (Eds.), *Contemporary research in adult and lifelong learning of mathematics (pp. 75-92)*. ICME-13 Monographs. Springer, Cham. https://doi.org/10.1007/978-3-319-96502-4_5

Cohen, S., Karimian Pour, C., & Kinsella, B. (2024). The persistence of mathematics: Integrating mathematical problem solving into the first-year engineering curriculum. *Proceedings of the Canadian Engineering Education Association*, 2025, Article 241. https://doi.org/10.24908/pceea.2024.18598

Dalby, D., & Noyes, A. (2016). Locating mathematics within post-16 vocational education in England. *Journal of Vocational Education & Training, 68*(1), 70–86. https://doi.org/10.1080/13636820.2015.1110828

Davor, I., Dawmah, P., & Larbi, E. (2026). An investigation into senior high school students' performance and difficulties in proving Euclidean geometry in mathematics curriculum: Evidence from Ghana. *Contemporary Mathematics and Science Education, 7*(1), Article ep26005. https://doi.org/10.30935/conmaths/17990

Dhakulkar, A., Olivier, J., & Bunt, B. J. (2023). A gamification approach to linear equations through creating and solving puzzles in a collaborative learning environment. In *M. Havenga, J. Oliver, & B. J. Bunt (Eds.), Problem-based learning and pedagogies of play* (pp. 141–166). AOSIS. https://doi.org/10.4102/aosis.2023.BK409.07

Firdaus, F., Akib, I., & Sukmawati, S. (2025). A conceptual framework for implementing a problem-solving based mathematics curriculum in vocational education. *PPSDP International Journal of Education*, *4*(2), 608–615. https://doi.org/10.59175/pijed.v4i2.766

FitzSimons, G. E. (2014). Commentary on vocational mathematics education: Where mathematics education confronts the realities of people's work. *Educational Studies in Mathematics, 86*(2), 291–305. https://doi.org/10.1007/s10649-014-9556-0

Folson, D., & Awuah, F. K. (2025). Assessing topic alignment in pre-tertiary TVET core mathematics: A Ghanaian case study (2011-2023). *Interdisciplinary Journal of Education, 8*(1), 35-56. https://doi.org/10.53449/yzxenc70.

Folson, D., & Awuah, F. K. (2026). Workplace experience learning at the pre-tertiary TVET level in Ghana: A phenomenological study. *SAGE Open, 16*(1), 1-16. https://doi.org/10.1177/21582440251411964

Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. Kluwer Academic Publishers. https://link.springer.com/book/9780792312994

Gbiang, P. B., & Attah, O. A. (2026). Technical, vocational education and training today: Challenges and prospects. *Journal of Technical and Vocational Education and Training, 1*(2). 22-33. https://doi.org/10.61955/LRCKVU

Guile, D., & Griffiths, T. (2001). Learning through work experience. *Journal of Education and Work, 14*(1), 113–131. https://doi.org/10.1080/13639080020028738

Hiebert, J., Gallimore, R., & Stigler, J. W. (2002). A knowledge base for the teaching profession: What would it look like and how can we get one? *Educational Researcher, 31*(5), 3–15. https://doi.org/10.3102/0013189X031005003

Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2013). Mathematics in the workplace: Issues and Challenges. In A. Damlamian, J. F. Rodrigues, & R. Strässer (Eds.), *Educational interfaces between mathematics and industry: Report on an ICMI-ICIAM-study* (pp. 43-50). (New ICMI study series; No. 16). Springer. https://doi.org/10.1007/978-3-319-02270-3_4

Jones, S.E. (2006). *Against technology: From the Luddites to Neo-Luddism (1st ed.)*. Routledge. https://doi.org/10.4324/9780203960455

Jones, K., & Tzekaki, M. (2016). Research on the teaching and learning of geometry. In *A. Gutiérrez, G. Leder, & P. Boero (Eds.), The second handbook of research on the psychology of mathematics education* (pp. 109-149), Sense Publishers. https://doi.org/10.1007/978-94-6300-561-6_4

Kaiser, G. (Ed.). (2017). *Proceedings of the 13th International Congress on Mathematical Education (ICME-13)*. Springer. https://doi.org/10.1007/978-3-319-62597-3

Kent, P., Noss, R., Guile, D., Hoyles, C., & Bakker, A. (2007). Characterizing the use of mathematical knowledge in boundary-crossing situations at work. *Mind, Culture, and Activity*, *14*(1-2), 64-82. https://doi.org/10.1080/10749030701307747

Kolb, D. A. (1984). *Experiential learning: Experience as the source of learning and development*. Englewood Cliffs, NJ: Prentice Hall. https://www.researchgate.net/publication/235701029_Experiential_Learning_Experience_As_The_Source_Of_Learning_And_Development

Kolb, D.A. (2015). *Experiential learning: Experience as the source of learning and development* (2nd ed.). Pearson Education. https://www.pearson.com/en-us/subject-catalog/p/experiential-learning-experience-as-the-source-of-learning-and-development/P200000000384/9780133892505?srsltid=AYCX9vlZfwo4msFxlRoy0rvGQeAqxa9Uzeo9K-Ajg4CaawNEONxAXH3d

Kyeremeh, P., Kwadwo Awuah, F., & Clark Orey, D. (2025). Challenges regarding the integration of ethnomathematical perspectives into geometry teaching: The faculty reflection. *Asian Journal for Mathematics Education*, *4*(3), 416-430. https://doi.org/10.1177/27527263251326330

Kwateng, F., & Leong, W. Y. (2025). Bridging tradition and innovation: Incorporating indigenous engineering practices into Ghana's secondary school curriculum. *INTI Journal*, *2025*(2), Article 9. https://doi.org/10.61453/INTIj.202521

Kwon, H., & Lee, Y. (2025). A meta-analysis of STEM project-based learning on creativity. *STEM Education*, *5*(2), 275–290. https://doi.org/10.3934/steme.2025014

Lakin, J. M., Wai, J., Olszewski-Kubilius, P., Corwith, S., Rothschild, D., & Uttal, D. H. (2024). Spatial thinking across the curriculum: Fruitfully combining research and practice. *Gifted Child Today*, *47*(3), 170-177. https://doi.org/10.1177/10762175241242722

Moore, A. M., Fairhurst, P., Bennett, J. M., Harrison, C., Correia, C. F., & Durk, J. (2024). Assessment and practical science: identifying generalizable characteristics of written assessments that reward and incentivise effective practices in practical science lessons. *International Journal of Science Education*, *46*(7), 643-669. https://doi.org/10.1080/09500693.2023.2253366

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic Inquiry*. SAGE Publications. https://www.sagepub.com/shop/buy-a-book/naturalistic-inquiry-1-842?srsltid=AYCX9vKGGk3BNd-70WJmGdjuVp1z9z5CM0uF6Q5XNNZg1Z6fgs5n_zfj

Madrilejos , K. (2024). Predicting academic performance in mathematics through online learning constructs and content proficiencies. *Journal of Interdisciplinary Perspectives*, *2*(5), 62–77. https://doi.org/10.69569/jip.2024.0065

Mayrhofer, J., Baldinger, S., Anđić, B., Riegler, V., & Weinhandl, R. (2026). Technology integration in physics education: using teachers' age, gender, and teaching experience to explore differences in cognitive engagement strategies. *Cogent Education, 13*(1), Article 2710501. https://doi.org/10.1080/2331186X.2026.2710501

Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). *Qualitative data analysis: A methods sourcebook* (3rd ed.). Sage Publications. https://eric.ed.gov/?id=ED565763

Mjenda, M., Mutarutinya, V., & Owiti, D. (2023). Diagnostic assessment knowledge and practices among secondary school mathematics teachers in Dodoma, Tanzania. *African Journal of Empirical Research*, *4*(2), 1119–1129. https://doi.org/10.51867/ajernet.4.2.114

Mlambo, P. B., & Sotsaka, D. T. S. (2025). Exploring synergies in Euclidean geometry and isometric drawing: A snapshot on grade 12 mathematics and engineering graphics & design. *Eurasia Journal of Mathematics, Science and Technology Education*, *21*(4), Article em2617. https://doi.org/10.29333/ejmste/16172

Motseki, P., & Jojo, Z. (2023). The effectiveness of using the geometer's sketchpad to enhance TVET college level 4 students' understanding of circle geometry. *Progressio, 43*, 1-17. https://doi.org/10.25159/2663-5895/12422

Moustakas, C. (1994). *Phenomenological research methods*. Sage. https://doi.org/10.4135/9781412995658

Ntereke, N., & Mphunyane, M. (2025). Perceptions of secondary school mathematics teachers and the challenges faced in implementing grade 9 curriculum: A case of two schools in Lesotho. *Journal of Mathematics and Science Teacher, 5*(2), Article em080. https://doi.org/10.29333/mathsciteacher/16060

Pantić, N. (2017). An exploratory study of teacher agency for social justice. *Teaching and Teacher Education, 66*, 219–230. https://doi.org/10.1016/j.tate.2017.04.008

Regnier, N. (2014). Informal learning in mathematics education. In S. Lerman (Eds.), *Encyclopedia of mathematics education* (289-295). Springer. https://doi.org/10.1007/978-94-007-4978-8_164

Saripudin, M., Hamdan, A. H., & Asiah, N. (2023). Indonesian Muslim students' perception on the concept of peace, characteristics of peaceful people, and non-peaceful experiences in Madrasah Aliyah. *Muslim Education Review*, *2*(1), 42–64. https://doi.org/10.56529/mer.v2i1.162

Sawyer, R. K. (2022). *The Cambridge handbook of the learning sciences (3rd ed.)*. Cambridge University Press. https://doi.org/10.1017/9781108888295

Schneider M, Beeres K, Coban L, Merz S, Susan Schmidt S, Stricker J, & De Smedt B. (2017). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: A meta-analysis. *Developmental Science, 20*(3), Article 12372. https://doi.org/10.1111/desc.12372

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information, 22*(2), 63-75. https://doi.org/10.3233/EFI-2004-22201

Singh, P., Md Nasir, N. A., & Sian Hoon, T. (2024). The cognitive gap in the mathematical thinking abilities of high school leavers for college: Are they ready? *Eurasia Journal of Mathematics, Science and Technology Education, 20*(11), Article em2529. https://doi.org/10.29333/ejmste/15577

Siswantari, Sabon, S. S., Listiawati, N., Wirda, Y., Zulkardi, & Riyanto, B. (2025). Bridging mathematics and communication: Implementing realistic mathematics education principles for skill development. *Journal on Mathematics Education*, *16*(2), 729-752. https://doi.org/10.22342/jme.v16i2.pp729-752

Smith, J. P. (1999). Tracking the mathematics of automobile production: Are schools failing to prepare students for work? *American Educational Research Journal, 36*(4), 835-878. https://doi.org/10.3102/00028312036004835

Sparrow, L. (2015). Mathematical knowledge for teaching at primary level. In S. J. Cho (Ed.), *The Proceedings of the 12th international congress on mathematical education (pp. 503-506)*. Springer. https://link.springer.com/chapter/10.1007/978-3-319-12688-3_48

Stolbova, I. D., K Gitman, Y., & Ovchinnikov, A. A. (2018, November). Integration of content and technologies of teaching within framework of geometrical-graphic training of students. *IOP Conference Series: Materials Science and Engineering*, 451, Article 012117. https://doi.org/10.1088/1757-899X%2F451%2F1%2F012117

Streefland, L. (2012). *Fractions in realistic mathematics education: A paradigm of developmental research (Vol. 8)*. Springer Science & Business Media. https://doi.org/10.1007/978-94-011-3168-1

Sunzuma, G., & Maharaj, A. (2019). Teacher-related challenges affecting the integration of ethnomathematics approaches into the teaching of geometry. *EURASIA Journal of Mathematics, Science and Technology Education, 15*(9), Article em1744. https://doi.org/10.29333/ejmste/100633

Susilo, B. E., Darhim, D., & Prabawanto, S. (2020). Critical thinking skills in integral calculus lecture based on mathematical dispositions. *Journal of Physics: Conference Series, 1521*(3), Article 032045. https://doi.org/10.1088/1742-6596/1521/3/032045

Tall, D.O. (2009). Dynamic mathematics and the blending of knowledge structures in the calculus. *ZDM Mathematics Education, 41*, 481–492 (2009). https://doi.org/10.1007/s11858-009-0192-6

Turner, J. C., & Meyer, D. K. (2000). Studying and understanding the instructional contexts of classrooms: Using our past to forge our future. *Educational Psychologist, 35*(2), 69–85. https://doi.org/10.1207/S15326985EP3502_2

Vimbelo, S. W., & Bayaga, A. (2023). Humanising pedagogy in mathematics education at South African Technical and vocational education and training (TVET) colleges: Influence on TVET teaching and learning. *International Journal of Learning, Teaching and Educational Research, 22*(9), 633-655. https://doi.org/10.26803/ijlter.22.9.34

Vollet, J. W., Kindermann, T. A., & Skinner, E. A. (2017). In peer matters, teachers matter: Peer group influences on students' engagement depend on teacher involvement. *Journal of Educational Psychology, 109*(5), 635–652. https://doi.org/10.1037/edu0000172

Vygotsky, L. S. (1978). *Mind in society: Development of higher psychological processes*. Harvard University Press. *https://www.hup.harvard.edu/books/9780674576292*

Wedege, T. (2010). People's mathematics in working life: Why is it invisible? *Adults Learning Mathematics: An International Journal, 5*(1), 89–97. https://files.eric.ed.gov/fulltext/EJ1068213.pdf

Xiong, X. (2024). Explore innovation and practice of collaborative training mode for vocational education teachers based on modern industrial college. *Helios Multidisciplinary, 1*(1), 1-23. https://doi.org/10.70702/bdb/snoz1817

Zhang, Y. (2025). Research on vocational education practice teaching mode under the background of "Internet plus". *International Journal of Web-Based Learning and Teaching Technologies, 20*(1),1- https://doi.org/10.4018/IJWLTT.382589

Zhou, D., Gomez, R., Wright, N., Rittenbruch, M., & Davis, J. (2022). A design-led conceptual framework for developing school integrated STEM programs: The Australian context. *International Journal of Technology and Design Education, 32*, 383–411. https://doi.org/10.1007/s10798-020-09619-5