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Nexus between research paradigm and mathematics education: An expository analysis

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ABSTRACT

The paper highlighted the nexus between various research paradigms vis-a-viz mathematics education. Research in education entails basing the process on three fundamental philosophical traditions – epistemology, ontology, and methodology. The trio formed the core of the paradigm likely to be employed to explain the research problem – in which case, mathematics education research problems. Thus, the paper argues that employing a research paradigm in mathematics education research has far-reaching effect in giving an expository analysis of the problem. The research paradigms that were discussed in this paper comprised of positivism, post-positivism, interpretivism and critical theory. In the end, the paper suggested that, in mathematics education, applying a given research paradigm in explaining a particular research problem is imperative, and can help in viewing the problem from philosophical perspective.

Keywords: mathematics, paradigm, nexus, expository, analysis

INTRODUCTION

The research paradigm denotes knowledge generation. In education and social sciences, there are different research paradigms employed when carrying out research; each with unique assumptions about what knowledge entails and how it is generated (Cohen et al., 2011). For instance, in education, research paradigms underpin one prominent feature, and that feature is "*incommensurability*" (Cohen et al., 2011, p. 5). By incommensurability, it means no two paradigms can occur at the same time in a particular research. Nevertheless, Turyahikayo (2021) and Hatch (2002) outlined the research paradigms mostly used in education research as positivism, post-positivism, interpretivism, critical theory, and post-modernism. Each of the paradigms is aligned to different ontological, epistemological as well as methodological inclination; depending on the context of the research. Thus, it is only when research anchors one of these research paradigms, the problem to be studied may not be properly explained. Therefore, in mathematics education, these research paradigms are equally used to explain the identified problem from ontological, epistemological and methodological perspectives respectively.

Now, the word 'paradigm' originates from the Greek word 'Paradeigma'. It implies '*pattern*'. The word was first used by Kuhn (1962) to indicate a research approach used by researchers at the time. Subsequently, the application of paradigm in research enabled researchers to develop a model expedient for scrutinizing problems in order to find solutions to such problems. Thus, Kuhn (1962) defined a paradigm as "*an integrated cluster of substantive concepts, variables, and problems attached with corresponding methodological approaches and tools.*" Similarly, Kuhn (1970) posited that research paradigm depicts certain beliefs, values and assumptions inherent in research culture – encompassing all research traditions from problem identification through setting context, determining subjects, methodological approach and results outcome. This culture is shared amongst researchers regarding the nature and conduct of the research. In line with this, Olsen et al. (1992) concluded that "*a paradigm implies a pattern, structure and framework or system of scientific and academic ideas, values and assumptions*" (p. 16).

Furthermore, paradigm connotes the mental picture of an idea or a systematic way of viewing an idea. Therefore, in general terms, paradigm can be referred to beliefs held by individuals to deal with ultimate reality as it unfolds within a given context. According to Guba and Lincoln (1994), paradigm represents the general philosophy of how individuals view the world and the nature of the possible relationships existing in that world. Similarly, the research paradigm chosen in a given study determines the methodology to be used in that study. It will also help to align the study with a particular philosophical orientation (Kivunja & Kuyini, 2017). In view of this, mathematics education research lends credence to these philosophical traditions by explaining the problems of teaching and learning from these lenses. For instance, according to Wahyuni (2012) interpretivists believe that "*reality is constructed by social actors and peoples' perceptions of it*" (p. 71). Meaning that interpretivism are of belief that people with different backgrounds, beliefs, assumptions, and experiences contribute to the construction of that reality unfolding in their social environments through social interactions – in which case, mathematics education as is required to be taught and learned.

However, this social reality can change arbitrarily depending upon the circumstances. This occurs due to human experiences, perceptions and perspectives that are naturally subjective. Interprevism allows for gaining deeper understanding of a phenomenon. Geertz (1994) call this "*thick description*" (p. 213) in interpretive paradigm. Thus, in interpretive paradigm, mathematics education knowledge has a place to be studied. In that, it will enable making sense of '*the why'* for the learners' mathematical knowledge acquisition, and the display of teachers' pedagogical competence, as well as provide an interpretation of such nexus between students' performance and teachers' pedagogical sophistication in an interpretive fashion.

Thus, scholars (Groth, 2010; Kalinowski et al., 2010) argue that qualitative approach to the study of mathematics provides more insights into the nature of the problem than quantitative by establishing pattern of relations or differences using a particular statistic only. For instance, as an aspect and/or an element of interpretivism, interviews reveal lines of thought not necessarily captured by hypothesis – which is key in quantitative research (Kalinowski et al., 2010). Similarly, Groth (2010) posited that

> Although qualitative methods have helped the fields of mathematics and statistics education move forward, and are still employed in current studies, the usefulness of qualitative studies is still frequently debated in political and scholarly discourse. Questions about qualitative research that were addressed during its rise to prominence among mathematics education researchers have re-surfaced in recent policy documents written by governmental agencies and scholars in the field. (p. 7).

Hence, the need for popularizing the application of research paradigm in mathematics educations.

Research Paradigm in Education

According to Guba and Lincoln (1994) the fundamental assumptions which define a particular research paradigm could be viewed by responses obtained through three (3) specific questions. The questions have to do with Ontology, Epistemology and Methodology. Thus, the first question borders on ontology and the question in respect of ontology is thus; *what is the nature of reality*? The second question is about epistemology – *what is the fundamental belief individuals held about knowledge and what can be known therefrom*? The third question is methodological in nature – *how can the researchers go about finding out what they think can be known from a given contextual problem*? In other words, "*how is knowledge gained*"? (Hatch, 2002, p. 12). Furthermore, research paradigms define for the researchers "*what is it they care about and what falls within and outside the limits of legitimate inquiry*" (Guba & Lincoln, 1994, p. 108). Every research paradigm is made up of four constituents elements, closely knitted to one another. They are: epistemology, ontology, axiology and methodology.

Research on Epistemology of Mathematics Education

Epistemology emanates from the Greek word *epistēmē,* meaning, 'knowledge' and *logos,* meaning 'logical discourse'. Therefore, epistemology is the philosophical assumption that has to do with the theory of knowledge. Epistemology studies the nature of knowledge, its justification and the rationality of belief(s). According to Steup and Zalta (2014), epistemology is based upon four fundamental beliefs. Firstly, philosophical analyses of the actual nature of knowledge and how it relates to concepts such as universal truthfulness, belief(s) and justification of the knowledge so acquired by the learner. Secondly, it deals with various problems of skepticism and "speculativism" of knowledge; thirdly, it serves as a source and scope of knowledge to be covered; and, fourthly, the criteria for establishing knowledge and its justification at rudimentary level. Thus, epistemology is the theory of the nature and grounds of knowledge as it concerns with the limit of knowledge and its validity (Burrell & Morgan, 2017). In this respect, it has to do with the nature, sources, origin and validity of knowledge and how human beings come to terms with the knowledge said to have been acquired.

Therefore, the justification for the need of epistemology in mathematics education is borne out of the desire for the researcher to explore the nature of mathematics knowledge, and how it can be taught from the lens of philosophy. Kenny and Fourie (2015) argued that epistemologically, there are three things every researcher(s) is intrinsically contending with in their research. They are: firstly, the relationship between the researcher and the researched. This type of relationship is viewed by Gunbayi (2020), and Ormston et al. (2014) as subjective, value-oriented, and considered as in the social world where the nexus between the researcher and the social phenomena unfolding is interactive. Secondly, the assumption about this relationship is "*that knowledge is essentially a relation between the learner and the phenomena being learned – between the knower and the known, the learner and the learned*" (Bolisani & Bratianu, 2018; Tooman et al., 2016). Thirdly, the assumption about how the knowledge is acquired – the methodology.

Now, some studies have been conducted regarding mathematics education epistemology; albeit with greater emphasis on beliefs (see Tamba & Cendana, 2021; Furinghetti, 2020; Ryan & Parra, 2019; Clark et al., 2018). In their study for instance, Tamba and Cendana (2021) examined the relationship between epistemological beliefs, teaching-learning beliefs and assessment beliefs in mathematics education. The study measured these three variables quantitatively and the results obtained indicated that there is positive correlation between beliefs on epistemology of mathematics and the beliefs on mathematics teaching and learning. Thus, epistemology in mathematics education studies underscores an essential part of the framework required to be reviewed with a view to providing a rich literature and coherent conversation of what the field entailed.

Ontology of Mathematics Education

On ontology, its need for being part of the research in mathematics education goes back to its philosophical position. Etymologically, the word "*ontology*" originates from the combination of two Greek words *onto,* which means "*being; that which is*" and *logos,* which means "to study". Therefore, ontology is the study of being. Philosophically, Klein (2003) opined that ontology is used to refer to the study of philosophical existence, or being. This type of study is geared towards the concept of being, through asking such questions as what 'being' means, or why does something 'exist'? It is also concerned with such type of question – what is the purpose of existence? According to Guba and Lincoln (1994), the ontological question "*what is the form and nature of reality, or what is there that can be known about it?*" (p. 108) is central to theoretical assumption about 'ontology' in mathematics education. In this regard, ontology concerns itself with the nature of reality – in which case mathematics education; beliefs 'about what' should be known and existence of 'what ought to be' known about the world. Such concepts as realism, materialism, idealism, pragmatism, and metaphysics are central to ontological perspectives of knowledge of mathematics.

Realism is the first ontological perspective. According to Cozzagilo (2021) realism is external to people's beliefs and their understanding – it exists as an independent entity. It means therefore, that the *understanding* of world as it exists differs from how individuals view the existence of the world. Therefore, research in Mathematics education has to be undergone to rectify these disparities in understanding as they unfold in the society. However, in materialism as the second ontological perspective, Turner (2022) claimed that there exist a real world but that only material appearance of it, such as economic relations, or physical features of that world, such as human interactions, are a reality. With this form of belief, materialism is strongly associated with Marxist philosophical school of thought. In idealism, there is believe that 'reality' is basically mental in nature; in that, it's mentally constructed, otherwise it is absolutely immaterial (Macionis, 2012; Randrup, 2002). Idealism put much emphasis on how the ideas of human beings, such as beliefs and values shape society. As for metaphysics, the last ontological perspective holds the beliefs about spiritual and immaterial world that human beings exist therein. It is concerned with things beyond human reasoning and imagination – supernatural occurrences within the cosmos.

Methodology of Mathematics Education

Methodology, as an aspect of a research paradigm and mathematics education is important. The methodological approach to research in mathematics education is two-fold – quantitative and qualitative. In qualitative methodology for instance, scholars like Edmund Husserl alluded to phenomenology to be employed as an interpretivists methodology (Namuwonge, 2024; Guo, 2024). Phenomenology is concerned with individuals' lived experiences. Husserlian phenomenology is subjective in nature and argues that truths about the reality of knowledge are embedded in human experiences (Vasterling, 2021; Qutoshi, 2018; Koopman, 2015). Therefore, it can be argued that research participants in mathematics education – who are the source of the essential truths in research, can be explored by the researcher from the participants' experiences. This methodology is applied in mathematics education research, as it focuses on obtaining the students' experiences and knowledge in learning mathematics. Now, various research paradigms have been discussed and their bearing to Mathematics education are succinctly explained.

POSITIVISM

Ontologically, positivists believe that objective and true reality exist in the universe independent of human experiences, beliefs and perceptions (Hatch, 2002). That, "*reality exists and driven by universal, natural laws*" (p. 12). Positivism assumes reality to be compartmentalized, that is, in separate entities. In which, each of the entity is capable of independent verification. After which, the components (entities) can be re-organized again.

This reality, however, cannot be characterized by time or context but it can be generalized. Epistemologically, positivists are concerned about "*what can be known, and what is the relationship of the knower to the known*"? (Hatch, 2002, p. 14). They claim that the universe has a specific pattern of existence and that pattern is discoverable through knowledge – Mathematics knowledge being one of those forms of knowledge. Positivists also claim to be objective in their quest for knowledge acquisition. As such, "researchers and the objects of their study are assumed to be mutually independent" in their research (p. 14). Because of this, positivists' researchers do not influence their subjects of study, nor are they influenced by the phenomenon they are studying. Methodologically, positivists are "experimental and manipulative" (Guba & Lincoln, 1994, p. 110) in nature. Their approach to research entails formulating questions or hypotheses that are to be tested empirically. In the end, their research findings are true only if they can be measured, verified and generalized (Voce, 2004). As a result, scientific approach to positivists' research paradigm is very pronounced. Therefore, research in mathematics education substantially depends on the provision and position of positivists regarding data collection, which can be measured, verified, and generalized quantitatively.

POST-POSITIVISM

Ontologically, there is some level of agreement between post-positivists and positivists. Both post-positivism and positivism agree that reality exists. But unlike positivists, post-positivists hold the belief that, "*because of the limitations of human inquiry, the inherent order of the universe can never be known completely*" (Hatch, 2002, p. 14). However, Cook and Campbell (1979) and Guba and Lincoln (1994) posit that post-positivists are more critical in their notion of reality than positivists. In that, they subject the claim about reality to a higher degree of scrutiny so as to increase the chances of anticipating the truths as closer as possible. Epistemologically, post-positivists try to establish as closer to the truth as possible in terms of knowledge acquisition. In view of this, Hatch (2002) argues that "*they seek to maintain an objective position in relation to the phenomena they are studying*" (p. 14). Thus, post-positivists researchers consider themselves as data collectors and employ the use of appropriate data research approaches like "*constant comparison*" (Glaser & Strauss, 1967) or "*analytic induction*" (Robinson, 1951, p. 812). This is to ensure that empirical data generated, not their personal imaginations or prejudices is collected. Methodologically, post-positivists use qualitative approach in their data collection process. Because they believe qualitative methods are more rigorous and enhance the level of validity and reliability of the research instruments. As such, there is relatively low inference from the data, logical and systematic procedures dominate the data analysis process (Glaser & Strauss, 1967).

INTERPRETIVISM

The weaknesses of the positivism led to the emergence of "interpretivism". Unlike the belief held by positivists, human beings are assumed to have freewill. Therefore, subjects should be studied as active agents in the research process so as to get meaningful interpretation of their actions as well as the underlying traits to be studied. According to Cohen and Manion (1994) the reason for the approach to interpretivism is to understand "*the world of human experience*" (p. 36). As such, interpretivism ontologically believes that "reality is socially constructed" (Mertens, 2005, p. 12). This is because absolute knowledge may not likely be possible (Powell, 2001). Elucidating more on this, Voce (2004) posited that interpretivism takes this world as a complex, dynamic and constructed entity in its outlook. In that, it is in academic writing interpreted and experienced by individuals in their interactions with one another and with a wide spectrum of social system. Thus, interpretivists hold the belief that reality is subjective and that individuals experience reality in their unique ways.

Epistemologically, interpretivists believe that knowledge is constructed. That, knowledge involves how individuals make meaning out of it and the bearing it has in their lives. To establish this knowledge, interpretivists do not generally start with a theory; instead, they "*generate or inductively develop a theory or*

pattern of meanings" (Creswell, 2012, p. 9) through their research process – and Mathematics is hinged on pattern formation in most of its aspects such as planes, constructions, interest rates, annuities etc. However, methodologically, interpretivists use such methods as unstructured observation, interviewing, discourse and content analysis (Reger & Kincaid, 2021; Mezmir, 2020; Ngulube, 2015). Interpretive research basically tries to capture "firsthand" information from the subjects. According to Charmaz (2014) this information (Mathematics knowledge[1]) from interpretive research perspective emanates from Grounded Theory. Therefore, mathematics education research within interpretive paradigm, uses test as the main methodology for investigating mathematical problems of, say, students' performance in learning mathematics and/or teachers' beliefs in using a particular teaching method in teaching.

CRITICAL THEORY

Critical Theory depicts an approach to the call for social engineering, entrenchment of Marxist's philosophy which focuses its attention on reflection, evaluation and critiquing of societal culture in order to checkmate the excesses of, and interrogate power structures to spearhead changes from status quo to something entirely different, new and equitable (Carrington & Selva, 2010; Kincheloe & McLaren, 2011). Its roots are traceable to sociology and literary text criticism. Proponents of critical theory argue that social upheaval sprang from social structures in the society and their cultural beliefs than from individuals themselves (Spector & Kitsuse, 2017). Similarly, it was argued that cultural ideology is one of the basic obstacles to human liberation and independence as part of the structures earlier mentioned that required to be interrogated (Ball et al., 2019). In view of this, when critical theory was introduced newly, it was considered to be within the realm of arts and humanities discipline. However, recently some scholars opined that it should be classified within a different category of discipline – away from social science – which is a subsidiary to arts and humanities.

Against the backdrop of this, critical theory is considered to be a "*school of thought"* practiced initially by the 'Frankfurt School' (Alexander, 2021; Abromeit, 2018; Bronner, 2013; Marcuse & Kellner, 2013). Therefore, Horkheimer in Abromeit (2018) described theory as critical in so far as it seeks "*to liberate human beings from the circumstances that enslave them*". This belief makes critical theory a category of modernism as well as postmodernism schools as well. Now, even though it is an offshoot of modernism, many experts in critical theory were skeptical of post-modernism. As a result, critical theory is considered to be one of the major components of both modern and post-modern 'schools of thought' and is widely used in arts and humanities (education being one of them) as well as other disciplines in social sciences today (Agger, 2012).

In addition to its origin in Frankfurt School, critical theory had been influenced by the works of Gyorgy Lukacs and Antonio Gramsci (Garlitz & Zompetti, 2023). Moreover, some Frankfurt School scholars have influenced the emergence of critical theory, notably Jurgen Habermas (Garlitz & Zompetti, 2023; Hanks, 2015). Similarly, in Habermas's work, it was believed to have originated from German idealism and flourished to becoming next to American pragmatism (Thompson, 2017). Due to the concern of critical theory "*base and superstructure*" for social settings, it is one of the remaining Marxist's philosophical ideas in current trends of social science scholarship (Felluga, 2015; Thompson, 2017; Bronner, 2013).

Furthermore, Stanford Encyclopedia of Philosophy tries to distinguish between "*Critical Theory*" (with emphasis on capitalization) being a result of decades of German philosophers and social scientists of the Frankfurt School, as well as much wider philosophical approaches that seek to emancipate human beings from the shackles of structural imbalances, and actively work to bring about drastic changes in the society according to human needs (usually called "*critical theory*", without emphasis on capitalization). Some philosophical terminologies deployed within this context include such concepts as the call for women

[1] The emphasis in the parenthesis is mine with respect to Mathematics knowledge; not emanating from the authors

emancipation (feminism), race versus identity crisis (critical race theory), and neo-colonialism (Gordon et al., 2019).

Noteworthy is that, Horkheimer defined critical theory first in his 1937 essay "*Traditional and Critical Theory*", as a socially-inclined theory aimed to critique and usher-in radical changes the society needs in its entirety, as against traditionally-inclined theory aimed only at understanding or explaining it. In his attempt to draw a line of distinction between *critical theory* – as a radically emancipating form of Marxist philosophy, Horkheimer went ahead to critique both models of science propounded by positivists. Thus, he described a theory to be critical in so far as it sought "*to liberate human beings from the circumstances that enslave them*". Hence, critical theory involves a sort of dimension in its uniqueness, either by criticizing societal norms and values, or by identifying anomalies inherent in society in terms of its own normative values and cultural heritage.

Postmodernism is another major aspect embedded in 'critical theory'. It analyses the polarization of cultural beliefs, affiliations and identities in order to challenge the ideas of modernists in their construction of identities such as meta-narratives, rationality of mind, and universality of truths, while bringing to the fore the political aspects of societal problems "*by situating them in historical and cultural contexts, to implicate themselves in the process of collecting and analyzing data, and to relativize their findings*". Proponents of critical theory have largely attributed it to Paulo Freire for its first application to education and particularly pedagogy (McKernan, 2013), considering the work he was best known for i.e. "*pedagogy of the Oppressed"*, which is a seminal work written in what is now known as the philosophy and social movement of 'critical pedagogy' in education. While dedicating the work to the marginalized segment of the society and based on his teaching experience in assisting Brazilian adults learners to acquire literacy and numeracy, Freire incorporated Marxist's ideas in his analysis and exposition of the relationship between the colonial masters and the natives (Rugut & Osman, 2013).

In the book, Freire addressed traditional pedagogy as what he called "*banking model of education*". This is because it considers students as hollow vessels (tabula rasa) to be filled with information – knowledge (Freire as cited in Alam, 2013). Freire further argued that pedagogy to be employed should consider learner as a co-developer of knowledge, not the other way round. However, different from banking model assertion, the teacher in this theory is not considered the transmitter of all knowledge alone, but a contributor who also learns with and from the students as well, even as they learn from the teacher himself (Gutiérrez, 2013). Thus, the ultimate aim of education is to emancipate learner from an oppressive belief of teacher versus student conundrum – a dichotomy synonymous with invader and the settler. Hence, it is not enough for students to analyse societal power dynamics and hierarchies through merely recognizing of imbalance and inequality. Thus, critical theory pedagogy must be seen to inculcate in learner the spirit of reflection and use that to challenge an oppressive power structures characterized by many educational institutions.

CONCLUSION

The paper highlighted the crux of research in mathematics education and brought to the fore the imperative of employing a particular research paradigm to explain a given research problem in Mathematics education. As one of the core fields of research in education, mathematics is robustly engaging and poses thought-provoking problems that require solid philosophical foundation to address. In order to achieve that, the application of research paradigm in viewing the problem from the lens of philosophy was highlighted. About four research paradigms were discussed and their bearing and/or nexus to mathematics problems was highlighted. Such paradigms as positivism, post-positivism, interpretivism and critical theory were discussed. In positivism, knowledge generation was viewed as a compartmentalized patterns of ideas that are capable of independent verification. Thus, in mathematics education, each aspect, whether philosophy, psychology, curriculum etc. has a unique form of approach to its study and is capable of being verified.

SUGGESTIONS

Firstly, mathematics teachers and researchers should be encouraged to explore philosophical ways of handling research data – both qualitative and quantitative with a view to making informed conclusions about the subjects of their research work. Failure to do so can result in committing errors that are avoidable. For instance, wrong usage of statistics and/or decision in quantitative research. Again, understanding research paradigm helps researchers in situating their research problem(s) within a given research paradigm that clearly explains the problem.

Secondly, when writing a book on mathematics education – whether on curriculum, psychology, philosophy, pedagogy etc., writers should be coherent in delineating their points and present their argument(s) in logical fashion. Lastly, policy makers have the leverage to implement what researchers have found in their research work. Thus, they have to thoroughly digest the findings so as to put them in proper perspective at the stage of implementation. When this happens, the said findings now becomes policy documents.

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